Numerical solution and FFT-based prediction of the hydrodynamic pressure generation of parallel rough surfaces

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Abstract - In general, the component macro-geometry (radius of curvature) determines the pressure generation and the roughness alters it somewhat. However, for parallel surfaces, the surface micro-geometry completely determines the hydrodynamic lubrication. This paper extends earlier work to numerically solve the hydrodynamic pressure generation and load carrying capacity (LCC) of surfaces with more complicated roughness features.

An FFT-based method is described to obtain the pressure distribution rapidly. The method is applicable to both measured surface topographies and artificially generated rough surfaces. Results show that it enables one to predict the hydrodynamic pressure, when cavitation is negligible. The relative error of the LCC over the central domain is smaller than 8\% while a 500\times time saving, compared to the numerical method, is obtained.

Keywords - hydrodynamic lubrication, parallel surfaces, load carrying capacity, FFT techniques

Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_m$</td>
<td>height amplitude of a sinusoidal surface</td>
</tr>
<tr>
<td>$c_l$</td>
<td>correlation length of surface</td>
</tr>
<tr>
<td>$d_{pm}$</td>
<td>pressure amplitude of sinusoidal surface</td>
</tr>
<tr>
<td>$d_{pmx}$</td>
<td>pressure amplitude of isotropic sinusoidal surface</td>
</tr>
<tr>
<td>$h$</td>
<td>film thickness</td>
</tr>
<tr>
<td>$H$</td>
<td>dimensionless film thickness, $H = h/h_0$</td>
</tr>
<tr>
<td>$h_0$</td>
<td>nominal film thickness (flying height)</td>
</tr>
<tr>
<td>$h_s$</td>
<td>roughness of the surface measured from the mean-height plane</td>
</tr>
<tr>
<td>$L_x$</td>
<td>domain length in $x$ direction</td>
</tr>
<tr>
<td>$L_y$</td>
<td>domain length in $y$ direction</td>
</tr>
<tr>
<td>$N_x$</td>
<td>number of grid points in $x$ direction</td>
</tr>
<tr>
<td>$N_y$</td>
<td>number of grid points in $y$ direction</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
</tr>
<tr>
<td>$P$</td>
<td>dimensionless pressure, $P = p/(12\eta u_m L_x/h_0^2)$</td>
</tr>
<tr>
<td>$p_a$</td>
<td>ambient pressure</td>
</tr>
<tr>
<td>$p_c$</td>
<td>cavitation pressure</td>
</tr>
<tr>
<td>$p_b$</td>
<td>boundary pressure, the difference between $p_a$ and $p_c$</td>
</tr>
<tr>
<td></td>
<td>and the cavitation pressure, $p_b = 3 \times 10^5$ [Pa]</td>
</tr>
<tr>
<td>$rms_h$</td>
<td>RMS roughness of surface</td>
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1. Introduction

Surface roughness influences the tribological performance of lubricated contacts, especially for cases of parallel surfaces, in terms of LCC, etc..

In 1966, Hamilton et al. [1] found that micro irregularities on the surfaces of rotary-shaft seals led to hydrodynamic pressure build-up. They observed local cavitation at the divergent part of the asperities by using a transparent rotor and proposed that it was the mechanism responsible for the observed load support. Different from the classical case where a single cavitation boundary occurs at the contact outlet, this kind of cavitation occurs inside the contact zone and is normally called inter-asperity cavitation [2, 3]. This inter-asperity cavitation has also been experimentally observed by Stakenborg [4], Qiu et al. [5] and Zhang et al. [6].

However, later studies have concluded that inter-asperity cavitation is not the sole source of load support [7, 8]. As stated in the review paper [9]: only in certain cases does it lead to additional load support, i.e. in fully textured contacts. Furthermore, in some cases, the load support may even be negative, depending on the shape of the asymmetric pressure distribution. For partially textured surfaces and certain cavitation pressure conditions, significant pressure can build up, leading to an important load support.

Accurate pressure distribution predictions require a mass-conservation treatment of cavitation, otherwise the cavitation area is underestimated [10, 11]. Interest in incorporating a mass-conserving cavitation algorithm into the numerical solution of hydrodynamic pressure of parallel surfaces, especially textured surfaces, has been substantial over the last two decades [9]. Examples can be found in [12, 13, 14, 15, 16] for parallel bearings, in [17, 18, 19, 20] for rotaory-shaft seals and in [21, 22, 23, 24] for piston ring-cylinder liner (PRCL) contacts.

As for the surface micro-asperities, surface roughness is generally considered random and in contrast, surface texturing describes well-defined features (discrete dimples, grooves) [9]. One of the first successful commercial applications of surface texturing is that of cylinder liners of internal combustion engines [25].

More recently, studies on the tribological performance of cylinder liners have been conducted by Tomanik et al. [26] for real surface topographies, by Yin et al. [27] for laser-textured surfaces and by Noutary et al. [28] for artificially-textured surfaces. Tomanik et al. [26] investigated the effect of waviness and roughness of the cylinder liner on the hydrodynamic and asperity pressures by using the measured topographies from two mirror-like coated bores. The simulation results reveal that most of the fluid pressure is generated by the honing grooves rather than by the localized pores on the coated surfaces. The two coated bore surfaces generate significantly higher hydrodynamic pressure and lower asperity contacts, compared with regular topographies. It is worth noting that these conclusions were obtained based on the same operating conditions of $h_0/rms$, not $h_e$ (see Notation section). A more in-depth discussion on the effect of dimples on lubrication can be found in [23, 28, 29]. Noutary et al. [28] found that partial texturing with micro dimples can induce positive pressure when the textured zone is located in the inlet.

Woloszynski et al. [30] developed an efficient algorithm, called Fischer-Burmeister-Newton-Schur (FBNS), for the joint solution of the Reynolds equation with mass-conserving cavitation and the Fischer-Burmeister equation for complementarity. They benchmarked the efficiency of the FBNS code which roughly yields two orders of magnitude reduction in computing time when compared against other algorithms, such as the augmented iterative Elrod-Adams, $p-\theta$, [31], the exact linear complementarity based on pivoting [32] and the modified switch function $g-g$ [33]. Biboulet et al. [34] developed an efficient global grid refinement solver which originates from the work of Woloszynski et al. [30], using the same mass-conserving cavitation algorithm as [28].

The solver by Biboulet et al. has shown a fast and stable convergence for parallel surfaces with sinusoidal roughness or regular dimple texturing. As an extension of [34], the current paper studies the hydrodynamic pressure of parallel surfaces with more complicated surface micro-structures. The authors used the cylinder bore surface topography, named "MLJ" in [26]. It is form-removed and then waviness-filtered.

Additionally, an FFT-based method is proposed to rapidly obtain the pressure distribution. The method is based on an approximate solution of sinusoidal roughness with standard perturbation techniques. Solutions of the perturbed Reynolds equation can be found in the work of Choy et al. [35] and Kim et al. [36]. The method is suitable to non-cavitation cases where the hydrodynamic lubrication problem remains approximately linear.

2. Theory

The main assumptions are those of a laminar flow regime under isothermal conditions. The dimensionless Reynolds equation with cavitation (mass-conserving) and the dimensionless Fischer-Burmeister equation are solved simultaneously:

$$\frac{\partial}{\partial x} \left( H^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( H^3 \frac{\partial p}{\partial y} \right) = \frac{\partial ((1-\theta)H)}{\partial x}$$ (1)

$$P + \theta - \sqrt{P^2 + \theta^2} = 0$$ (2)
where $H$ is the dimensionless gap; $P$ is the dimensionless pressure; $X$ is the dimensionless sliding direction; $Y$ is perpendicular to $X$; $\theta$ is the cavitation fraction. Eq.(2) indicates the complementarity: $\theta = 0$ and $P > 0$; $0 < \theta < 1$ and $P = 0$.

Using finite difference techniques, these equations are discretized and a local Jacobian linearisation is used. A grid refinement strategy is used to quickly converge the cavitation boundaries. The implementation details and the validation of the numerical solver are presented in [34].

For evaluating the relative central mean pressure, we define

$$LCC/(0.25L_x L_y) = \sum_{N_x/2}^{N_x} \sum_{N_y/2}^{N_y} p(x, y)/ (0.25N_x N_y) - p_0$$

using the LCC over the central domain. $p_0$, the boundary pressure, is the difference between the ambient pressure and the cavitation pressure; $L_x$ ($L_y$) is the length of the domain in the $x$ ($y$) direction; $N_x$ ($N_y$) is the number of the discrete points in the $x$ ($y$) direction.

3. Numerical results

As an example, we solved the hydrodynamic pressure of a cylinder liner - oil control ring (OCR) using the numerical method. The moving ring surface is considered to be smooth.

The stationary surface is measured from a coated cylinder bore surface, called "MLJ" in [26]. The surface size is $L_x \times L_y = 0.8 \times 0.8 \text{ mm}^2$ and the RMS roughness is $0.073$ $\mu$m. The operating conditions are listed in Table 1. We solved two cases with $h_0 = 2 \mu$m and $h_0 = 1 \mu$m.

The number of points on the finest grid is $N_x \times N_y = 1024 \times 1024$. The hydrodynamic pressure results are shown in Figure 1. For the first case (Figure 1(b)), no cavitation occurs.

<table>
<thead>
<tr>
<th>Table 1: Operating conditions</th>
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</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Oil viscosity, $\eta$</td>
</tr>
<tr>
<td>Mean surface velocity, $u_m$</td>
</tr>
<tr>
<td>Sliding direction,</td>
</tr>
<tr>
<td>Boundary pressure, $p_0$</td>
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</tbody>
</table>

With the smooth OCR surface moving forwards ($+x$ direction), the grooves of the rough cylinder liner suck in oil, which leads to a low pressure zone inside the grooves and slightly to its left. Cavitation occurs if the pressure drops below the cavitation pressure. In Figure 1(c), the pressure variation becomes larger due to the relatively larger variation of the geometry $h_0 = 1 \mu$m. Behind the grooves, high pressure is built up, which is clearly visible for the biggest groove. This is in accordance with the description by Tomanik et al. [26]: most of the hydrodynamic pressure was generated by the honing grooves rather than by the localized pores. The hydrodynamic pressure rise tends to occur at the convergent portion of the honing grooves and extends onto the smooth plateau region behind the grooves.

As in [28], we know that the pressure build-up is dependent on the length of the plateau: the larger the length, the higher the pressure.

By comparing the pressurized (full-film) zone behind the biggest groove, one finds that the local pressure in the second case with $h_0 = 1 \mu$m is much higher than for $h_0 = 2 \mu$m. As explained by Harp and Salant [3], the pressure elevation is a result of the near-by inter-asperity cavitation.

But this does not mean that the LCC in the second case (with cavitation) is higher. The calculation results of the average pressure $LCC/(0.25L_x L_y)$ are $-9.8 \times 10^3$ Pa for $h_0 = 1 \mu$m and $5.43 \times 10^3$ Pa for $h_0 = 2 \mu$m.

Therefore, as concluded by Gropper et al. [9] through a review of many papers on the hydrodynamic lubrication of textured surfaces, cavitation cannot and should not be named as the responsible load support mechanism and the influence of cavitation highly depends on the operating conditions.

The hydrodynamic-lubrication performance (pressure build-up, cavitation, LCC, etc.) of parallel surfaces is influenced not only by the surface structure, the flying height but also by the difference of the ambient pressure and the cavitation pressure.

The parameter $h_0/rms.h$ shows the relative importance of roughness effects [37]. From the work by Harp et al. [2], one also knows that $h_0/rms.h$ and $p_0$ are the key indicators for cavitation. With other operating conditions being constant, as both $h_0/rms.h$ and $p_0$ decrease, the extent of cavitation is expected to increase. In this paper, $p_0 = 3 \times 10^4$ Pa is used for all calculations. For the pressure distribution without cavitation (Figure 1(b)), $h_0/rms.h$ = 27.4 is used, while for the pressure distribution with cavitation (Figure 1(c)), $h_0/rms.h$ = 13.7 is used. Please note that the pressure scale of Figure 1(c) is exactly twice the scale of Figure 1(b)!

When $h_0$ is much larger than the $rms.h$, the surface roughness has a weak effect and grooves play a less dominant role. Consequently, cavitation disappears.
Figure 1: Cylinder liner surface: (a) height distribution, (b) pressure at \( h_0 = 2 \, \mu m \) and (c) pressure at \( h_0 = 1 \, \mu m \). Note: cavitation is represented by the white areas.

The computing times are respectively 194 s and 243 s with a very small residual error up to \( 1 \times 10^{-10} \). As a grid refinement strategy is used, the calculation for cavitating cases only needs slightly more time to converge [34]. In order to analyse the numerical accuracy, we solved the first case \( (h_0 = 2 \, \mu m) \) on two other grids as. The LCC results are shown in Table 2.

<table>
<thead>
<tr>
<th>Grid: ( N_x \times N_y )</th>
<th>( LCC/(0.25L_x L_y) )</th>
<th>Computing time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>512 \times 512</td>
<td>5.57 \times 10^3</td>
<td>43</td>
</tr>
<tr>
<td>1024 \times 1024</td>
<td>5.43 \times 10^3</td>
<td>184</td>
</tr>
<tr>
<td>2048 \times 2048</td>
<td>5.32 \times 10^3</td>
<td>1291</td>
</tr>
</tbody>
</table>

The difference between the solutions on the coarser grids is 3%. That for the finer grids is 2%. The computing time on grid 1024 \times 1024 is roughly 4 times that on grid 512 \times 512. But the time consumption increase for grid 2048 \times 2048 is considerable. To pursue fast calculation, we propose an efficient pressure-prediction method based on perturbation techniques and FFT. It is precise for cases where cavitation plays a minor role.

4. Pressure prediction

4.1. Approximate solution using perturbation techniques

The film thickness is given by \( h(x, y) = h_0 - h_r(x, y) \), where \( h_r \) is the surface roughness function. When the surface amplitude is small, \( h_r \) can be seen as a perturbation term. The function \( h \) in the left-hand side of the Reynolds equation (Eq.(1) in dimensional form) can be approximated by \( h_0 \). Then for non-cavitation cases where \( \theta = 0 \), one gets

\[
\frac{\partial}{\partial x} (h_0 \frac{\partial p}{\partial x}) + \frac{\partial}{\partial y} (h_0 \frac{\partial p}{\partial y}) = 12 \eta u_m \frac{\partial h_0}{\partial x} \tag{4}
\]

A first-order pressure approximation can be written as \( p = p_0 + \epsilon p_1 \). The solution \( p_0 \) is easily obtained because for smooth parallel surfaces, the pressure is constant everywhere and equal to the boundary pressure. Introduction of the perturbed film thickness and pressure into Eq.(4) results in:

\[
\epsilon \left( \frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} \right) = \frac{-12 \eta u_m \partial h_0}{h_0^2} \tag{5}
\]

We analyse the effect of a sinusoidal surface roughness, described as:

\[
h_r(x, y) = Amp \sin \left( \frac{2\pi}{\lambda_x} x + \phi_x \right) \sin \left( \frac{2\pi}{\lambda_y} y + \phi_y \right) \tag{6}
\]

where \( Amp \) is the roughness amplitude; \( \lambda_x (\lambda_y) \) is the wavelength; \( \phi_x (\phi_y) \) is the phase.

One defines the perturbation parameter \( \epsilon = Amp/h_0 \). Eq.(5) is then transformed into

\[
\frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} = -\frac{24 \eta u_m}{\lambda_x h_0^2} \cos \left( \frac{2\pi}{\lambda_x} x + \phi_x \right) \sin \left( \frac{2\pi}{\lambda_y} y + \phi_y \right) \tag{7}
\]

\[
p_1 = \frac{6}{\pi} \frac{\lambda_x}{1 + (\lambda_x/\lambda_y)^2} \frac{\eta u_m}{h_0^2} \sin \left( \frac{2\pi}{\lambda_x} x + \phi_x + \frac{\pi}{2} \right) \sin \left( \frac{2\pi}{\lambda_y} y + \phi_y \right) \tag{8}
\]
So, the approximate pressure solution,
\[ p = p_0 + \epsilon p_1, \]
with the pressure amplitude being
\[ dpm = \frac{6}{\pi} \frac{\lambda_x \eta u_m \text{Amp}}{h_0} \quad (10) \]
Eq.(9) also indicates that the predicted LCC of a sinusoidal surface is 0. Eq.(10) reveals that whether cavitation occurs is dependent not only on \( h_0 \) and \( \text{rms}_h \) (=Amp/2 for a sinusoidal surface) but also on the surface wavelength and the operating conditions (the lubricant viscosity and mean surface velocity). If \( dpm < p_0 \), there is no cavitation.

4.2. Verification of the approximate solution
For an isotropic sinusoidal roughness (\( \lambda_y = \lambda_x \)), the pressure amplitude is
\[ dpm = \frac{3}{\pi} \frac{\eta u_m \lambda_x \text{Amp}}{h_0} \quad (11) \]
We numerically solved the pressure of isotropic sinusoidal surfaces under the conditions of \( 1.5 \leq h_0 \leq 2.5 \) [\( \mu \text{m} \)], \( 0.01 \leq \text{Amp} \leq 0.1 \) [\( \mu \text{m} \)], \( 2 \leq L_x / \lambda_x \leq 30 \). The operating conditions are the same as in Table 1. The surface size is \( L_x \times L_y = 2 \text{ mm} \times 2 \text{ mm} \). The calculations were performed on a grid with 1024 \( \times \) 1024 points. Figure 3 shows that the pressure amplitude is indeed linear in the term \( \lambda_x \text{Amp}/h_0^3 \) and Eq.(11) gives a good fit with an R-square of 0.998.

Figure 3: Pressure amplitude as a function of Amp.

With Eqs.(10,11), one gets
\[ \frac{dpm}{dpm_x} = \frac{2}{1 + (\lambda_x/\lambda_y)^2} \quad (12) \]
Numerical calculations were performed for \( 5 \leq L_x / \lambda_x \leq 20 \), \( 0.1 \leq \lambda_x / \lambda_y \leq 10 \), \( 20 \leq h_0 / \text{Amp} \leq 40 \).
Figure 4 shows a decreasing trend of $d\pi / d\pi_x$ with $\lambda_x / \lambda_y$ and the relationship is very well fitted by Eq.(12).

![Graph showing decreasing trend of $d\pi / d\pi_x$ with $\lambda_x / \lambda_y$.]

**4.3. Pressure prediction with FFT techniques**

According to Fourier analysis, a general surface can be represented by the sum of simple sine waves. Since the perturbed Reynolds equation (5) is linear, the solution consists of a sum of sinusoidal pressure components.

We use FFT techniques to predict the hydrodynamic pressure of measured or artificially generated surfaces. The flow chart of this method is shown in Figure 5.

Please note that 'Amplitude relationship' refers to Eq.(10). A phase shift of $\pi/2$ in the x direction is used for 'Phase shift'. This procedure can be easily implemented using MATLAB software [38]. The details are given in the Appendix.

**4.4. Results**

We use random surfaces to study the pressure and LCC for hydrodynamic lubrication of parallel surfaces, which can represent seals, parallel bearings or PRCL contacts.

These surfaces were generated with a Gaussian height distribution and an exponential autocorrelation function according to the algorithm outlined by Patir [39]. Here, all the generated surfaces have the same correlation length $c = 0.2 \text{ mm}$ (decaying to 10%) in both the x and y directions and are periodical with a period of $[0, L_x]$ in the x direction and $[0, L_y]$ in the y direction.

Figure 6 shows such a surface roughness and its power spectral density (PSD).

![Flow chart of the FFT-based pressure prediction.]  

![Figure 6: (a) Artificially generated random rough surface and (b) power spectral density.]
The 2-dimensional (2-D) PSD is defined according to [40] as:

$$PSD(p, q) = |c_{p,q}|^2 (N_xN_y)^2 / (L_xL_y). \quad (13)$$

The domain $L_x \times L_y$ is 2 mm $\times$ 2 mm. The RMS roughness $r_{ms\_h}$ is 0.01 µm. $h_0$ is equal to 2 µm and the values of $\eta, u_m$ and $p_0$ are listed in Table 1. These parameters ensure that no cavitation occurs.

The wavelengths of the sinusoidal components from the Fourier decomposition are most likely to be smaller than the correlation length of the surface. Using the parameter values $\lambda_0=c_l=0.2$ mm, $Amp=0.05$ µm and $h_0=2$ µm in Eq.(11), the calculation result is $dp_{m_x}=6250$ Pa, much smaller than $p_0 (=30000$ Pa).

We both predicted and numerically solved the pressure with the same finest discretization of $N_x \times N_y = 1024 \times 1024$ points. The pressure results are shown in Figure 7.

Figure 7: Top-view of the pressure distribution of the generated rough surface: (a) numerical, (b) FFT-prediction, (c) numerical result in the central domain and (d) FFT prediction in the central domain.
Once again, it is found that the predicted pressure distribution near the boundaries differs substantially from the numerical result, but the central pressure distribution results are very close. The numerical average pressure is $LCC/(0.25L_xL_y) = -578$ Pa. The relative error of the LCC over the central domain is about 5%. This shows that the FFT method cannot predict the LCC over the total domain but can predict the LCC over the central area accurately. The time consumption of the numerical calculation and the FFT-based prediction are respectively 346 s and 0.7 s. So, we obtained a time saving of roughly $500\times$ with the FFT method.

Additionally, we conducted a comparison of the numerical solution and FFT-based prediction for 10 generated random surfaces under the same parameter conditions.

The results are presented in Figure 8. It shows that for all the calculations, the relative error of the LCC over the central domain is smaller than 8%. Furthermore, the LCC is randomly positive or negative. Generally, for parallel surfaces, no cavitation leads to no load carrying capacity [28], which means that theoretically, the LCC should be 0.

The average LCC over the central domain of 10 random surfaces is -990 Pa. Those of 40 surfaces and 90 surfaces are respectively -520 Pa and -360 Pa. According to the "Law of large numbers", the average LCC ratio of 40 to 10 surfaces should be 1/2 and that of 90 to 10 surfaces 1/3. It can be seen that the numerical ratios are cloase. This shows that the average LCC of a large sample tends to 0, which is in accordance with observations from [28]. It should be noted that the FFT mean pressure prediction over the total domain is zero by definition (Eq. (9)).

5. Discussion

5.1. Influence of the cavitation pressure

Figure 9 shows the calculated pressure distributions of the measured surface "MLJ" (Figure 1(a)) for different cavitation pressures $p_c$. The boundary pressure $p_w$ is the difference between the ambient pressure ($p_a$) and the cavitation pressures ($p_c$). One assumes $p_a$ is 100 kPa. So from (a) to (f), $p_w$ takes the values of 0, 5, 10, 15, 20, 30 [kPa]. As a result, the cavitation area covers 49.0%, 13.5%, 4.6%, 1.0%, 0.07%, 0.0% and the mean pressure $LCC/(0.25L_xL_y)$ is 0.9, -1.7, 0.2, 3.5, 5.3, 5.4 [kPa].

One finds that the cavitation area decreases with the decrease of the cavitation pressure. When cavitation is severe, the LCC is very small and even negative. As $p_c$ gets smaller than 85 kPa, no cavitation occurs and the LCC is constant.

5.2. Application to non-Gaussian roughness

The prediction method is not limited to Gaussian roughness or periodic surfaces. Figure 10 shows the comparison between the predicted pressure and the numerical result (the same as Figure 1(b)). The "MLJ" surface is measured and neither Gaussian nor periodic.

It can be seen that the pressure difference near the boundaries is relatively large, as the Dirichlet condition $p = p_w$ makes the problem non-periodic. Neglecting the pressure near the boundaries, it is found that the predicted pressure distribution is similar to the numerical one. The relative error of the LCC over the central area is 8.0% and a time saving of $500\times$ is obtained with the FFT-based method.
Figure 9: Top view of the pressure distribution of the "MLJ" surface for different cavitation pressure $p_{\text{c}}$: 100, 95, 90, 85, 80, 70 [kPa] from (a) to (f). $h_0=2 \mu m$ other conditions as in Table 1. please note the different color bars for top and bottom rows.

Figure 10: Top-view of the pressure distribution of the measured "MLJ" topography: numerical (left) and FFT-prediction (right).

5.3. Accuracy of the prediction method
The prediction method is based on two assumptions: $\text{rms}_s/h_0 \ll 1$ and no cavitation. Figure 11 shows the variation of the prediction accuracy with the ratio $\text{rms}_s/h_0$. The LCC error refers to the error of the mean pressure $LCC/(0.25 L_x L_y)$. The pressure error is defined as the ratio $\text{RMS}((\text{pressure difference})/\text{Mean}(\text{pressure})$ over the central domain. We used the same random roughness to obtain the surfaces by only changing the $\text{rms}_h$ value. One finds that the pressure error increases with the ratio $\text{rms}_s/h_0$. The error is 2.3% at $\text{rms}_s/h_0=0.01$. The LCC error shows a more complicated trend but it remains smaller than 8% for
\[ \frac{\text{rms}_h}{h_0} \leq 0.01. \] When the ratio is larger than 0.01, cavitation occurs and both the pressure and LCC errors rise rapidly. However, the critical value of \( \frac{\text{rms}_h}{h_0} \) depends on other operating conditions, i.e. the cavitation pressure.

According to the analysis in Section 5.1, one knows that a larger \( p_c \) is more likely to result in cavitation which makes the prediction method less accurate. Consequently, the critical value of \( \frac{\text{rms}_h}{h_0} \) may be smaller than 0.01.

The numerical results show that in non-cavitation cases, the LCC is almost constant in \( p_c \) (\( p_0 \)). The predicted LCC depends on other operating conditions and the surface feature. So, the cavitation pressure, \( p_c \), does not influence the accuracy when there is no cavitation.

6. Conclusions

The current paper studies the hydrodynamic pressure and LCC of parallel surfaces, which can be piston ring-cylinder liner contacts, rotatory-shaft seals or parallel bearings. Reference solutions are generated using an existing solver.

For non-cavitating cases where the problem becomes approximately linear, an efficient pressure-prediction method is proposed, using FFT techniques.

This prediction is based on a perturbation solution of the non-cavitation hydrodynamic lubrication of stationary sinusoidal surfaces with a moving smooth surface. Neglecting the pressure near the boundaries, the pressure distribution is sinusoidal as well. A linear relationship is found between the pressure amplitude of isotropic sinusoidal surfaces and the term \( \text{Amp} \frac{\lambda_p}{h_0} \). Subsequently, the hydrodynamic pressure in the centre of anisotropic sinusoidal surfaces can be predicted using the obtained relationship between the pressure amplitude ratio (\( \frac{dp_{m}}{dp_{m,x}} \)) and the wavelength ratio (\( \frac{\lambda_p}{\lambda_y} \)).

Good agreement is found between the numerical solution and the FFT-based prediction of the central hydrodynamic pressure and central LCC for the measured surface topography and artificial surfaces with random roughness. The FFT prediction method results in a 500x time saving.

The LCC of an artificially generated random surface can be positive or negative, depending on the precise roughness features. The average LCC of a large sample of random surfaces is 0.

The ratio of \( h_0 \) and \( \text{rms}_h \) determines the level of cavitation, when the waviness feature of the surface and the operating conditions (the lubricant viscosity and mean surface velocity) are constant. When the ratio is large, no cavitation occurs. A significant increase in the local pressure (built up behind the valleys) is found for a cavitating case, compared to a non-cavitation case.

So far, the FFT-based method could not successfully predict the hydrodynamic pressure with cavitation. Future work will study the prediction of the pressure distribution with severe cavitation.

6.1. Acknowledgements

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7. References


8. Appendix

Implementation of the pressure prediction method in MATLAB software using the commands 'fft', 'ifft', 'angle' and 'abs':

\[ c_{p,q} = \frac{fft(h(x,y))}{(N_x \cdot N_y)} \]

\[
\begin{align*}
\text{abs}(c_{p,q}) &= \begin{cases} 
Amp_{p,0} & p = 0, q = 0 \\
0.5Amp_{p,q^*} & p = 0, q \neq 0 \text{ or } q = 0, q \neq 0 \\
0.25Amp_{p^*,q^*} & p \neq 0, q \neq 0 
\end{cases} \\
\text{angle}(c_{p,q}) &= sgn(p)(\varphi_{xp^*} - 0.5\pi) + sgn(q)(\varphi_{yp^*} - 0.5\pi)
\end{align*}
\]

where sgn(x) is the sign function.

\[
\text{abs}(dpm_{p,q}) = \text{Fun}(q^*/p^*) \text{abs}(c_{p,q}) \frac{\eta u_m L_x}{(p^* h_0^2)}, \quad \lambda_x = L_x/p^* \\
\text{angle}(dpm_{p,q}) = \text{angle}(c_{p,q}) \quad \text{(with replacing } \varphi_{xp^*} \text{ by } \varphi_{xp^*} + 0.5\pi) \\
p(x, y) = ifft(dpm_{p,q})N_xN_y + p_0
\]